June 20, 2011

WA Web Appendix

WA.1 Regrouped Expressions

$N^{-1}T^{-1}\boldsymbol{\Phi}'\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{F}$ has a generic element that can be written

$$
\begin{aligned}
&N^{-1}T^{-1}\sum_{i,t}\phi_i(m_1)F_t(m_2)\varepsilon_{it} \\
&\quad - T^{-1/2}\left(N^{-1}\sum_i \phi_i(m_1)\right)\left(N^{-1}\sum_j T^{-1/2}\sum_t F_t(m_2)\varepsilon_{jt}\right) \\
&\quad - T^{-1/2}\left(T^{-1}\sum_s F_s(m_2)\right)\left(N^{-1}\sum_j T^{-1/2}\sum_t \phi_j(m_1)\varepsilon_{jt}\right) \\
&\quad + N^{-1/2}T^{-1/2}\left(T^{-1}\sum_s F_s(m_2)\right)\left(N^{-1}\sum_i \phi_i(m_1)\right)\left(N^{-1/2}T^{-1/2}\sum_{j,t}\varepsilon_{jt}\right).
\end{aligned}
$$

$N^{-1}T^{-1/2}\boldsymbol{\Phi}'\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{\omega}$ has a generic element that can be written

$$
\begin{aligned}
&N^{-1}T^{-1/2}\sum_{i,t}\phi_i(m_1)\omega_t(m_2)\varepsilon_{it} \\
&\quad - \left(N^{-1}\sum_i \phi_i(m_1)\right)\left(N^{-1}\sum_j T^{-1/2}\sum_t \omega_t(m_2)\varepsilon_{jt}\right) \\
&\quad - \left(T^{-1}\sum_s \omega_s(m_2)\right)\left(N^{-1}T^{-1/2}\sum_{j,t} \phi_j(m_1)\varepsilon_{jt}\right) \\
&\quad + N^{-1/2}T^{-1/2}\left(T^{-1/2}\sum_s \omega_s(m_2)\right)\left(N^{-1}\sum_i \phi_i(m_1)\right)\left(N^{-1/2}T^{-1/2}\sum_{j,t}\varepsilon_{jt}\right).
\end{aligned}
$$

$N^{-1}T^{-1/2}\boldsymbol{\Phi}\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{\eta}$ has a generic element that can be written

$$
\begin{aligned}
&N^{-1}T^{-1/2}\sum_{i,t}\phi_i(m)\varepsilon_{it}\eta_{t+h} \\
&\quad - N^{-1/2}\left(T^{-1}\sum_t N^{-1/2}\sum_i \phi_i(m)\varepsilon_{it}\right)\left(T^{-1/2}\sum_s \eta_{s+h}\right) \\
&\quad - N^{-1/2}\left(N^{-1}\sum_i \phi_i(m)\right)\left(N^{-1/2}T^{-1/2}\sum_{jt}\varepsilon_{jt}\eta_{t+h}\right) \\
&\quad + N^{-1/2}T^{-1/2}\left(N^{-1}\sum_i \phi_i(m)\right)\left(N^{-1/2}T^{-1/2}\sum_{j,t}\varepsilon_{jt}\right)\left(T^{-1/2}\sum_s \eta_{s+h}\right).
\end{aligned}
$$

$N^{-1}T^{-3/2}\boldsymbol{F}'\boldsymbol{J}_T\boldsymbol{\varepsilon}\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{F}$ has a generic element that can be written

$$
\begin{aligned}
&N^{-1}T^{-3/2}\sum_{i,s,t}F_s(m_1)\varepsilon_{is}\varepsilon_{it}F_t(m_2)\\
&\quad-\left(T^{-1}\sum_t N^{-1}T^{-1/2}\sum_{i,s}F_s(m_1)\varepsilon_{is}\varepsilon_{it}\right)\left(T^{-1}\sum_u F_u(m_2)\right)\\
&\quad-\left(T^{-1}\sum_s F_s(m_1)\right)\left(T^{-1}\sum_t N^{-1}T^{-1/2}\sum_{i,u}\varepsilon_{it}\varepsilon_{iu}F_u(m_2)\right)\\
&\quad+\left(T^{-1}\sum_s F_s(m_1)\right)\left(T^{-1}\sum_u N^{-1}T^{-1/2}\sum_{i,t}\varepsilon_{it}\varepsilon_{iu}\right)\left(T^{-1}\sum_v F_v(m_2)\right)\\
&\quad-T^{-1/2}\left(N^{-1}\sum_i T^{-1/2}\sum_s F_s(m_1)\varepsilon_{is}\right)\left(N^{-1}\sum_j T^{-1/2}\sum_t \varepsilon_{jt}F_t(m_2)\right)\\
&\quad+N^{-1/2}T^{-1/2}\left(N^{-1}\sum_i T^{-1/2}\sum_s F_s(m_1)\varepsilon_{is}\right)\left(N^{-1/2}T^{-1/2}\sum_{j,t}\varepsilon_{jt}\right)\left(T^{-1}\sum_u F_u(m_2)\right)\\
&\quad+N^{-1/2}T^{-1/2}\left(T^{-1}\sum_s F_s(m_1)\right)\left(N^{-1/2}T^{-1/2}\sum_{i,t}\varepsilon_{it}\right)\left(N^{-1}\sum_j T^{-1/2}\sum_u \varepsilon_{ju}F_u(m_2)\right)\\
&\quad-N^{-1}T^{-1/2}\left(T^{-1}\sum_s F_s(m_1)\right)\left(N^{-1/2}T^{-1/2}\sum_{i,t}\varepsilon_{it}\right)\left(N^{-1/2}T^{-1/2}\sum_{j,u}\varepsilon_{ju}\right)\left(T^{-1}\sum_v F_v(m_2)\right).
\end{aligned}
$$

$N^{-1}T^{-1/2}\boldsymbol{F}'\boldsymbol{J}_T\boldsymbol{\varepsilon}\boldsymbol{J}_N\boldsymbol{\varepsilon}_t$ has a generic element that can be written

$$
\begin{aligned}
&N^{-1}T^{-1/2}\sum_{i,s}F_s(m)\varepsilon_{is}\varepsilon_{it}\\
&\quad-N^{-1/2}\left(N^{-1}\sum_i T^{-1/2}\sum_s F_s(m)\varepsilon_{is}\right)\left(N^{-1/2}\sum_j \varepsilon_{jt}\right)\\
&\quad-\left(T^{-1}\sum_s F_s(m)\right)\left(N^{-1}T^{-1/2}\sum_{i,u}\varepsilon_{iu}\varepsilon_{it}\right)\\
&\quad N^{-1}\left(T^{-1}\sum_s F_s(m)\right)\left(N^{-1/2}T^{-1/2}\sum_{i,u}\varepsilon_{iu}\right)\left(N^{-1/2}\sum_j \varepsilon_{jt}\right).
\end{aligned}
$$

$N^{-1}T^{-3/2}\boldsymbol{\eta}'\boldsymbol{J}_T\boldsymbol{\varepsilon}\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{F}$ has a generic element that can be written

$$
\begin{aligned}
&T^{-1/2}\left(N^{-1}T^{-1}\sum_{i,s,t}F_s(m)\varepsilon_{is}\varepsilon_{it}\eta_{t+h}\right)\\
&\quad - T^{-1/2}\left(T^{-1}\sum_t N^{-1}T^{-1/2}\sum_{i,s}F_s(m)\varepsilon_{is}\varepsilon_{it}\right)\left(T^{-1/2}\sum_u \eta_{u+h}\right)\\
&\quad - \left(T^{-1}\sum_s F_s(m)\right)\left(N^{-1}T^{-3/2}\sum_{i,t,u}\varepsilon_{it}\varepsilon_{iu}\eta_{u+h}\right)\\
&\quad + T^{-1/2}\left(T^{-1}\sum_s F_s(m)\right)\left(T^{-1}\sum_u N^{-1}T^{-1/2}\sum_{i,t}\varepsilon_{it}\varepsilon_{iu}\right)\left(T^{-1/2}\sum_v \eta_{v+h}\right)\\
&\quad - N^{-1/2}T^{-1/2}\left(N^{-1}\sum_i T^{-1/2}\sum_s F_s(m)\varepsilon_{is}\right)\left(N^{-1/2}T^{-1/2}\sum_{t,j}\varepsilon_{jt}\eta_{t+h}\right)\\
&\quad + N^{-1/2}T^{-1}\left(N^{-1}\sum_i T^{-1/2}\sum_s F_s(m)\varepsilon_{is}\right)\left(N^{-1/2}T^{-1/2}\sum_{j,t}\varepsilon_{jt}\right)\left(T^{-1/2}\sum_u \eta_{u+h}\right)\\
&\quad + N^{-1}T^{-1/2}\left(T^{-1}\sum_s F_s(m)\right)\left(N^{-1/2}T^{-1/2}\sum_{i,t}\varepsilon_{it}\right)\left(N^{-1/2}T^{-1/2}\sum_{j,u}\varepsilon_{ju}\eta_{u+h}\right)\\
&\quad - N^{-1}T^{-1/2}\left(T^{-1}\sum_s F_s(m)\right)\left(N^{-1/2}T^{-1/2}\sum_{i,t}\varepsilon_{it}\right)\left(N^{-1/2}T^{-1/2}\sum_{j,u}\varepsilon_{ju}\right)\left(T^{-1}\sum_v \eta_{v+h}\right).
\end{aligned}
$$

WA.2 plim of $\hat{\boldsymbol{\beta}}$

$\hat{\boldsymbol{\beta}}_3 = N^{-2}T^{-3}\boldsymbol{Z'J}_T\boldsymbol{XJ}_N\boldsymbol{X'J}_T\boldsymbol{XJ}_N\boldsymbol{X'J}_T\boldsymbol{Z}$ can be written

$$
\begin{aligned}
N^2T^3\hat{\boldsymbol{\beta}}_3 = {} & \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\omega} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Lambda'} + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\omega}.
\end{aligned}
$$

$\hat{\boldsymbol{\beta}}_4 = N^{-1}T^{-2}\boldsymbol{Z'J}_T\boldsymbol{XJ}_N\boldsymbol{X'J}_T\boldsymbol{y}$ can be written

$$
\begin{aligned}
NT^2\hat{\boldsymbol{\beta}}_4 = {} & \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\beta} + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\eta} + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\beta} + \boldsymbol{\Lambda F'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\eta} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\beta} + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\eta} + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\beta} + \boldsymbol{\omega'J}_T\boldsymbol{F\Phi'J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\eta} \\
& + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{F\beta} + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi F'J}_T\boldsymbol{\eta} + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\beta} + \boldsymbol{\Lambda F'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\eta} \\
& + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi FJ}_T\boldsymbol{F\beta} + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\Phi FJ}_T\boldsymbol{\eta} + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{F\beta} + \boldsymbol{\omega'J}_T\boldsymbol{\varepsilon J}_N\boldsymbol{\varepsilon'J}_T\boldsymbol{\eta}.
\end{aligned}
$$

In the expression for $N^2T^3\hat{\boldsymbol{\beta}}_3$ we see the following terms, whose probability limits we show:

Lemma Web *Let Assumptions 1-4 hold. Then we have the following as $N, T \to \infty$:*

1. $N^{-2}T^{-1}\boldsymbol{\Phi}'\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{\varepsilon}\boldsymbol{J}_N\boldsymbol{\Phi} = \boldsymbol{O}_p(T^{-1/2}\delta_{NT}^{-1})$
2. $N^{-2}T^{-2}\boldsymbol{\Phi}'\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{\varepsilon}\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{F} = \boldsymbol{O}_p(N^{-1/2}\delta_{NT}^{-1})$
3. $N^{-2}T^{-2}\boldsymbol{\Phi}'\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{\varepsilon}\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{\omega} = \boldsymbol{O}_p(N^{-1/2}\delta_{NT}^{-1})$
4. $N^{-2}T^{-3}\boldsymbol{F}'\boldsymbol{J}_T\boldsymbol{\varepsilon}\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{\varepsilon}\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{F} = \boldsymbol{O}_p(T^{-1}\delta_{NT}^{-2})$
5. $N^{-2}T^{-3}\boldsymbol{F}'\boldsymbol{J}_T\boldsymbol{\varepsilon}\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{\varepsilon}\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{\omega} = \boldsymbol{O}_p(T^{-1}\delta_{NT}^{-2})$
6. $N^{-2}T^{-3}\boldsymbol{\omega}'\boldsymbol{J}_T\boldsymbol{\varepsilon}\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{\varepsilon}\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{\omega} = \boldsymbol{O}_p(T^{-1}\delta_{NT}^{-2})$

Proof:

First, we note that the argument for $N^{-1}T^{-1/2}\sum_{i,s} F_s(m)\varepsilon_{is}\varepsilon_{it}$ in Lemma 1 can be adapted to show that

$$N^{-1/2}T^{-1}\sum_{i,t}\phi_i(m)\varepsilon_{it}\varepsilon_{jt} = O_p(\delta_{NT}^{-1})$$

by using Assumption 2.2.

Item 1 is $K \times K$ with generic (m_1, m_2) element

$$\begin{aligned}
&N^{-2}T^{-1}\sum_{i,j,t}\phi_i(m_1)\varepsilon_{it}\varepsilon_{jt}\phi_j(m_2) - N^{-3}T^{-1}\sum_{i,j,k,t}\phi_i(m_1)\varepsilon_{it}\varepsilon_{jt}\phi_k(m_2)\\
&\quad - N^{-3}T^{-1}\sum_{i,j,k,t}\phi_i(m_1)\varepsilon_{jt}\varepsilon_{kt}\phi_k(m_2) + N^{-4}T^{-1}\sum_{i,j,k,l,t}\phi_i(m_1)\varepsilon_{jt}\varepsilon_{kt}\phi_l(m_2)\\
&\quad - N^{-2}T^{-2}\sum_{i,j,s,t}\phi_i(m_1)\varepsilon_{is}\varepsilon_{jt}\phi_j(m_2) + N^{-3}T^{-2}\sum_{i,j,k,s,t}\phi_i(m_1)\varepsilon_{is}\varepsilon_{jt}\phi_k(m_2)\\
&\quad + N^{-3}T^{-2}\sum_{i,j,k,s,t}\phi_i(m_1)\varepsilon_{js}\varepsilon_{kt}\phi_k(m_2) - N^{-4}T^{-2}\sum_{i,j,k,l,s,t}\phi_i(m_1)\varepsilon_{js}\varepsilon_{kt}\phi_l(m_2)\\
&\quad 1.\text{I} - \cdots - 1.\text{VIII}.
\end{aligned}$$

Adapting the argument for $N^{-1}T^{-3/2}\boldsymbol{F}'\boldsymbol{J}_T\boldsymbol{\varepsilon}\boldsymbol{J}_N\boldsymbol{\varepsilon}'\boldsymbol{J}_T\boldsymbol{F}$ in Lemma 2, we have that: 1.I is $O_p(N^{-1})$; 1.II-IV are $O_p(N^{-1/2}\delta_{NT}^{-1})$; 1.V is $O_p(N^{-1})$; 1.VI-VII are $O_p(N^{-1}T^{-1/2})$; and 1.VIII is $O_p(N^{-3/2}T^{-1/2})$. Summing we have that Item 1 is $\boldsymbol{O}_p(N^{-1/2}\delta_{NT}^{-1})$.

Item 2 is $K \times K$ with generic (m_1, m_2) element

$$\begin{aligned}
&N^{-2}T^{-2}\sum_{i,j,s,t}\phi_j(m_1)\varepsilon_{jt}\varepsilon_{it}\varepsilon_{is}F_s(m_2) - N^{-2}T^{-3}\sum_{i,j,s,t,u}\phi_j(m_1)\varepsilon_{ju}\varepsilon_{iu}\varepsilon_{it}F_s(m_2)\\
&\quad - N^{-3}T^{-2}\sum_{i,j,k,s,t}\phi_k(m_1)\varepsilon_{kt}\varepsilon_{jt}\varepsilon_{is}F_s(m_2) + N^{-3}T^{-3}\sum_{i,j,k,s,t,u}\phi_k(m_1)\varepsilon_{ku}\varepsilon_{ju}\varepsilon_{it}F_s(m_2)\\
&\quad - N^{-2}T^{-3}\sum_{i,j,s,t,u}\phi_j(m_1)\varepsilon_{ju}\varepsilon_{it}\varepsilon_{is}F_s(m_2) + N^{-2}T^{-4}\sum_{i,j,s,t,u,v}\phi_j(m_1)\varepsilon_{jv}\varepsilon_{iu}\varepsilon_{it}F_s(m_2)\\
&\quad + N^{-3}T^{-3}\sum_{i,j,k,s,t,u}\phi_k(m_1)\varepsilon_{ku}\varepsilon_{jt}\varepsilon_{is}F_s(m_2) - N^{-3}T^{-4}\sum_{i,j,k,s,t,u,v}\phi_k(m_1)\varepsilon_{kv}\varepsilon_{ju}\varepsilon_{it}F_s(m_2)\\
&\quad - N^{-3}T^{-2}\sum_{i,j,k,s,t}\phi_k(m_1)\varepsilon_{jt}\varepsilon_{it}\varepsilon_{is}F_s(m_2) + N^{-3}T^{-3}\sum_{i,j,k,s,t,u}\phi_k(m_1)\varepsilon_{ju}\varepsilon_{iu}\varepsilon_{it}F_s(m_2)\\
&\quad + N^{-4}T^{-2}\sum_{i,j,k,l,s,t}\phi_l(m_1)\varepsilon_{kt}\varepsilon_{jt}\varepsilon_{is}F_s(m_2) - N^{-4}T^{-3}\sum_{i,j,k,l,s,t,u}\phi_l(m_1)\varepsilon_{ku}\varepsilon_{ju}\varepsilon_{it}F_s(m_2)\\
&\quad + N^{-3}T^{-3}\sum_{i,j,k,s,t,u}\phi_k(m_1)\varepsilon_{ju}\varepsilon_{it}\varepsilon_{is}F_s(m_2) - N^{-3}T^{-4}\sum_{i,j,k,s,t,u,v}\phi_k(m_1)\varepsilon_{jv}\varepsilon_{iu}\varepsilon_{it}F_s(m_2)\\
&\quad - N^{-4}T^{-3}\sum_{i,j,k,l,s,t,u}\phi_l(m_1)\varepsilon_{ku}\varepsilon_{jt}\varepsilon_{is}F_s(m_2) + N^{-4}T^{-4}\sum_{i,j,k,l,s,t,u,v}\phi_l(m_1)\varepsilon_{kv}\varepsilon_{ju}\varepsilon_{it}F_s(m_2)\\
&\quad 2.\mathrm{I} - \cdots - 2.\mathrm{XVI}.
\end{aligned}$$

2.I is bounded by

$$N^{-1/2}\delta_{NT}^{-1}\left(N^{-1}\sum_i[N^{-1/2}T^{-1}\delta_{NT}\sum_{j,t}\phi_j(m_1)\varepsilon_{jt}\varepsilon_{it}]^2\right)^{1/2} T^{-1/2}\left(N^{-1}\sum_i[T^{-1}\sum_{s,t}F_s(m_2)F_t(m_2)\varepsilon_{is}\varepsilon_{it}]^2\right)^{1/2}$$

Both objects within brackets are $O_p(1)$, therefore both objects within parentheses are $O_p(1)$ and it follows that 2.I is $O_p(N^{-1/2}T^{-1/2}\delta_{NT}^{-1})$.

2.II is bounded by

$$N^{-1/2}\delta_{NT}^{-1}\left(N^{-1}\sum_i[N^{-1/2}T^{-1}\delta_{NT}\sum_{j,u}\phi_j(m_1)\varepsilon_{ju}\varepsilon_{iu}]^2\right)^{1/2} T^{-1/2}\left(N^{-1}\sum_i[T^{-3}\sum_{s,t,u,v}F_s(m_2)F_t(m_2)\varepsilon_{iu}\varepsilon_{iv}]\right)$$

Both objects within brackets are $O_p(1)$, therefore both objects within parentheses are $O_p(1)$ and it follows that 2.II is $O_p(N^{1/2}T^{-1/2}\delta_{NT}^{-1/2})$.

2.III can be written

$$N^{-1/2}\delta_{NT}^{-1}\left(N^{-1}\sum_j[N^{-1/2}T^{-1}\delta_{NT}\sum_{k,t}\phi_k(m_1)\varepsilon_{kt}\varepsilon_{jt}]\right)\left(N^{-1}\sum_i[T^{-1}\sum_s F_s(m_2)\varepsilon_{is}]\right)$$

Both objects with brackets are $O_p(1)$, therefore both objects within parentheses are $O_p(1)$ and it follows that 2.III is $O_p(N^{-1/2}\delta_{NT}^{-1})$.

Notice that in every item in Lemma 2 and Item 1 here, the generic element's first, second or third term has dictated the stochastic order of the entire item. In the interest of space, we note that this fact is true here and therefore state that terms 2.III-2.XVI are of no larger order than 2.III. Thus Item 2 is $\boldsymbol{O}_p(N^{-1/2}\delta_{NT}^{-1})$.

Item 3 is $K \times M$ and follows from the above arguments by substituting $w_s(m_2)$ for $F_s(m_2)$.

Item 4 is $K \times K$ with generic (m_1, m_2) element

$$
\begin{aligned}
&N^{-2}T^{-3}\sum_{i,j,s,t,u} F_u(m_1)\varepsilon_{ju}\varepsilon_{jt}\varepsilon_{it}\varepsilon_{is}F_s(m_2) - N^{-2}T^{-4}\sum_{i,j,s,t,u,v} F_v(m_1)\varepsilon_{jt}\varepsilon_{ju}\varepsilon_{iu}\varepsilon_{it}F_s(m_2) \\
&\quad - N^{-3}T^{-3}\sum_{i,j,k,s,t,u} F_u(m_1)\varepsilon_{ku}\varepsilon_{kt}\varepsilon_{jt}\varepsilon_{is}F_s(m_2) + N^{-3}T^{-4}\sum_{i,j,k,s,t,u,v} F_v(m_1)\varepsilon_{kv}\varepsilon_{ku}\varepsilon_{ju}\varepsilon_{it}F_s(m_2) \\
&\quad - N^{-2}T^{-4}\sum_{i,j,s,t,u,v} F_v(m_1)\varepsilon_{jv}\varepsilon_{ju}\varepsilon_{it}\varepsilon_{is}F_s(m_2) - N^{-2}T^{-5}\sum_{i,j,s,t,u,v,w} F_w(m_1)\varepsilon_{jw}\varepsilon_{jv}\varepsilon_{iu}\varepsilon_{it}F_s(m_2) \\
&\quad + N^{-3}T^{-4}\sum_{i,j,k,s,t,u,v} F_v(m_1)\varepsilon_{kv}\varepsilon_{ku}\varepsilon_{jt}\varepsilon_{is}F_s(m_2) - N^{-2}T^{-5}\sum_{i,j,k,s,t,u,v,w} F_w(m_1)\varepsilon_{kw}\varepsilon_{kv}\varepsilon_{ju}\varepsilon_{it}F_s(m_2) \\
&\quad - N^{-3}T^{-3}\sum_{i,j,k,s,t,u} F_u(m_1)\varepsilon_{ku}\varepsilon_{jt}\varepsilon_{it}\varepsilon_{is}F_s(m_2) + N^{-3}T^{-4}\sum_{i,j,k,s,t,u,v} F_v(m_1)\varepsilon_{kv}\varepsilon_{ju}\varepsilon_{iu}\varepsilon_{it}F_s(m_2) \\
&\quad + N^{-4}T^{-3}\sum_{i,j,k,l,s,t,u} F_u(m_1)\varepsilon_{lu}\varepsilon_{kt}\varepsilon_{jt}\varepsilon_{is}F_s(m_2) - N^{-4}T^{-4}\sum_{i,j,k,l,s,t,u,v} F_v(m_1)\varepsilon_{lv}\varepsilon_{ku}\varepsilon_{ju}\varepsilon_{it}F_s(m_2) \\
&\quad + N^{-3}T^{-4}\sum_{i,j,k,s,t,u,v} F_v(m_1)\varepsilon_{kv}\varepsilon_{ku}\varepsilon_{ju}\varepsilon_{it}F_s(m_2) - N^{-3}T^{-5}\sum_{i,j,k,s,t,u,v,w} F_w(m_1)\varepsilon_{kw}\varepsilon_{jv}\varepsilon_{iu}\varepsilon_{it}F_s(m_2) \\
&\quad - N^{-4}T^{-4}\sum_{i,j,k,l,s,t,u,v} F_v(m_1)\varepsilon_{lv}\varepsilon_{ku}\varepsilon_{jt}\varepsilon_{is}F_s(m_2) + N^{-4}T^{-5}\sum_{i,j,k,l,s,t,u,v,w} F_w(m_1)\varepsilon_{lw}\varepsilon_{kv}\varepsilon_{ju}\varepsilon_{it}F_s(m_2) \\
&\quad - N^{-2}T^{-4}\sum_{i,j,s,t,u,v} F_v(m_1)\varepsilon_{ju}\varepsilon_{jt}\varepsilon_{it}\varepsilon_{is}F_s(m_2) + N^{-2}T^{-5}\sum_{i,j,s,t,u,v,w} F_w(m_1)\varepsilon_{jv}\varepsilon_{ju}\varepsilon_{iu}\varepsilon_{it}F_s(m_2) \\
&\quad + N^{-3}T^{-4}\sum_{i,j,k,s,t,u,v} F_v(m_1)\varepsilon_{ku}\varepsilon_{kt}\varepsilon_{jt}\varepsilon_{is}F_s(m_2) - N^{-3}T^{-5}\sum_{i,j,k,s,t,u,v,w} F_w(m_1)\varepsilon_{kv}\varepsilon_{ku}\varepsilon_{ju}\varepsilon_{it}F_s(m_2) \\
&\quad + N^{-2}T^{-5}\sum_{i,j,s,t,u,v,w} F_w(m_1)\varepsilon_{jv}\varepsilon_{ju}\varepsilon_{it}\varepsilon_{is}F_s(m_2) - N^{-2}T^{-6}\sum_{i,j,s,t,u,v,w,x} F_x(m_1)\varepsilon_{jw}\varepsilon_{jv}\varepsilon_{iu}\varepsilon_{it}F_s(m_2) \\
&\quad - N^{-3}T^{-5}\sum_{i,j,k,s,t,u,v,w} F_w(m_1)\varepsilon_{kv}\varepsilon_{ku}\varepsilon_{jt}\varepsilon_{is}F_s(m_2) + N^{-3}T^{-6}\sum_{i,j,k,s,t,u,v,w,x} F_x(m_1)\varepsilon_{kw}\varepsilon_{kv}\varepsilon_{ju}\varepsilon_{it}F_s(m_2) \\
&\quad + N^{-2}T^{-4}\sum_{i,j,k,s,t,u,v} F_v(m_1)\varepsilon_{kv}\varepsilon_{ju}\varepsilon_{it}\varepsilon_{is}F_s(m_2) - N^{-3}T^{-5}\sum_{i,j,k,s,t,u,v,w} F_w(m_1)\varepsilon_{kv}\varepsilon_{ju}\varepsilon_{iu}\varepsilon_{it}F_s(m_2) \\
&\quad - N^{-4}T^{-4}\sum_{i,j,k,l,s,t,u,v} F_v(m_1)\varepsilon_{lu}\varepsilon_{kt}\varepsilon_{jt}\varepsilon_{is}F_s(m_2) + N^{-4}T^{-5}\sum_{i,j,k,l,s,t,u,v,w} F_w(m_1)\varepsilon_{lv}\varepsilon_{ku}\varepsilon_{ju}\varepsilon_{it}F_s(m_2) \\
&\quad - N^{-3}T^{-5}\sum_{i,j,k,s,t,u,v,w} F_w(m_1)\varepsilon_{kv}\varepsilon_{ju}\varepsilon_{it}\varepsilon_{is}F_s(m_2) + N^{-3}T^{-6}\sum_{i,j,k,s,t,u,v,w,x} F_x(m_1)\varepsilon_{kw}\varepsilon_{jv}\varepsilon_{iu}\varepsilon_{it}F_s(m_2) \\
&\quad + N^{-4}T^{-5}\sum_{i,j,,k,l,s,t,u,v,w} F_w(m_1)\varepsilon_{lv}\varepsilon_{ku}\varepsilon_{jt}\varepsilon_{is}F_s(m_2) - N^{-4}T^{-6}\sum_{i,j,k,l,s,t,u,v,w,x} F_x(m_1)\varepsilon_{lw}\varepsilon_{kv}\varepsilon_{ju}\varepsilon_{it}F_s(m_2) \\
&\quad = 4.\mathrm{I} + \cdots - 4.\mathrm{XXXII}.
\end{aligned}
$$

4.I is bounded by

$$
T^{-1}\delta_{NT}^{-2}\left(T^{-1}\sum_t [N^{-1}T^{-1/2}\delta_{NT}\sum_{j,u} F_u(m_1)\varepsilon_{ju}\varepsilon_{jt}]^2\right)^{1/2}\left(T^{-1}\sum_t [N^{-1}T^{-1/2}\delta_{NT}\sum_{i,s} F_s(m_2)\varepsilon_{is}\varepsilon_{it}]^2\right)^{1/2}
$$

Both objects within brackets are $O_p(1)$, therefore both objects within parentheses are $O_p(1)$ and it follows that 4.I is $O_p(T^{-1}\delta_{NT}^{-2})$.

4.II can be written

$$
N^{-1}T^{-1}\left(T^{-1}\sum_v F_v(m_1)\right)\left(N^{-1}T^{-1}\sum_{i,j,t,u} \varepsilon_{jt}\varepsilon_{ju}\varepsilon_{iu}\varepsilon_{it}\right)\left(T^{-1}\sum_s F_s(m_1)\right)
$$

All objects within parentheses are $O_p(1)$, the second one because the expectation of its absolute value is finite by Assumption 2 and the triangle inequality. It follows that 4.II is $O_p(N^{-1}T^{-1}$.

4.III is bounded by

$$N^{-1/2}T^{-1}\delta_{NT}^{-1}\left(T^{-1}\sum_t [N^{-1/2}\sum_j \varepsilon_{jt}]^2\right)^{1/2}\left(T^{-1}\sum_t [N^{-1}T^{-1/2}\delta_{NT}^{-1}\sum_{k,u} F_u(m_1)\varepsilon_{kt}\varepsilon_{kt}]^2\right)^{1/2}$$
$$\left(N^{-1}\sum_i [T^{-1/2}\sum_s F_s(m_2)\varepsilon_{is}]\right)$$

All objects within brackets are $O_p(1)$, therefore all objects within parentheses are $O_p(1)$ and it follows that 4.III is $O_p(N^{-1/2}T^{-1}\delta_{NT}^{-1})$. These terms dictate the item's order, and therefore Item 4 is $\boldsymbol{O}_p(T^{-1}\delta_{NT}^{-2})$.

Item 5 is $K \times M$ and follows from the above arguments by substituting $w_t(m_2)$ for $F_t(m_2)$; Item 6 is $M \times M$ and follows from Item 5 by substituting $w_s(m_1)$ for $F_s(m_1)$.

QED